Lecture 9

Conditional CAPM

The CAPM Revisited

• Let's rewrite the CAPM DGP:

$$R_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t} (R_{m,t} - r_f) + \varepsilon_{i,t}$$
$$\beta_i = Cov(R_{i,t}, R_{m,t})/Var(R_{m,t})$$

• The CAPM can be written in terms of cross sectional returns. That is the SML:

$$E[R_{i,t} - r_f] = \gamma_0 + \gamma_1 \beta_i$$

There is a linear constant relation between $E[R_{i,t} - r_f]$ and β_i.

• This version of the CAPM is called the *static* CAPM, since β_i is constant, or *unconditional CAPM*, since conditional information plays no role in determining excess returns.

• Q: Is beta unresponsive to (conditioning) information?

• Suppose that in January we have information about asset i's next dividend. Suppose this was true for every stock. Then, what should the risk/return tradeoff look like over the course of a year?

• Time-varying expected returns are possible.

• Q: What about time-varying risk premia?

• Other problems with an unconditional CAPM:
  – Leverage causes equity betas to rise during a recession (affects asset betas to a lesser extent).
  – Firms with different types of assets will be affected by the business cycle in different ways.
  – Technology changes.
  – Consumers' tastes change.
  – One period model, with multi-period agents.

• In particular, the unconditional CAPM does not describe well the CS of average stock returns: The SML fails in the CS.

• The CAPM does not explain why, over the last forty years:
  - small stocks outperform large stocks (the "size effect").
  - firms with high book-to-market (B/M) ratios outperform those with low B/M ratios (the "value premium").
  -stocks with high prior returns during the past year continue to outperform those with low prior returns ('momentum').

The Conditional CAPM

• We have discussed a lot of anomalies that reject CAPM. Recall that some of the "anomaly" variables seemed related to β.

• Simple idea ("trick") to "rescue" the CAPM: The 'anomaly' variables proxy for time-varying market risk exposures:

$$R_{i,t} - r_f = \alpha_{i,t} + \beta_{i,t} (R_{m,t} - r_f) + \varepsilon_{i,t}$$
$$\beta_{i,t} = Cov_t(R_{i,t},R_{m,t})/Var_t(R_{m,t}) = Cov\,(R_{i,t},R_{m,t}|I_t)/Var_t(R_{m,t|}I_t)$$

where I_t represents the information set available at time t. (Note, the conditional cross-sectional CAPM notation used I_{t-1} to represent the information set available at time t. Accordingly, they also use $\beta_{i,t-1}$.

=> $\beta_{i,t-1}$ is time varying. Conditional information can affect $\beta_{i,t-1}$.

• In the SML formulation of the CAPM (and using usual notation):

$$R_{i,t} - r_f = \gamma_{0,t-1} + \gamma_{1,t-1}\, \beta_{i,t-1} + \varepsilon_{i,t}$$

• The SML is used to explain CS returns. Taking expectations:

$$E[R_{i,t} - r_f] = E[\gamma_{0,t-1}] + E[\gamma_{1,t-1}]\, E[\beta_{i,t-1}] + Cov(\gamma_{1,t-1},\beta_{i,t-1})$$

If the $Cov(\gamma_{1,t-1},\beta_{i,t-1})=0$ (or a linear function of the expected beta) for asset i, then we have the static CAPM back: expected returns are a linear function of the expected beta.

• In general, $Cov(\gamma_{1,t-1},\beta_{i,t-1})\neq 0$. During bad economic times, the expected market risk premium is relatively high, more leveraged firms are likely to face more financial difficulties and have higher conditional betas.

=> Given I_{t-1}, Cov($\gamma_{1,t-1}$,$\beta_{i,t-1}$) = 0 is testable.

• This is the base for conditional CAPM testing.

• Q: But, what is the right conditioning information set, I_{t-1}?
Usually, papers condition on observables.

- Estimation error and Roll's critique are still alive.
- If the variables in I_{t-1} are chosen according to previous research, data mining problems are also alive and well.

• Q: How do we model $\beta_{i,t-1}$ –actually, how do we model Cov($\gamma_{1,t-1}$,$\beta_{i,t-1}$)?
A great source of papers. The conditional CAPM is an ad-hoc attempt to explain anomalies. (Moreover, in general, theory does not tells us much about functional forms or conditioning variables.)

=> It us up to the researchers to come up with $\beta_{i,t-1}= f(Z_{t-t})$

• There are two usual approaches to model $\beta_{i,t-1}$:

(1) Time-series, where the dynamics of $\beta_{i,t-1}$ are specified by a time series model.

(2) Exogenous driving variables: $\beta_{i,t-1}= f(Z_t)$, where Z_t is an exogenous variable (say D/P, size, etc.). In general, f(.) is linear.

Example: $\beta_{i,t-1} = \beta_{i,0} + \beta_{i,1} Z_t$

$$R_{i,t} = \alpha_i + (\beta_{i,0} + \beta_{i,1} Z_t) R_{m,t} + \varepsilon_{i,t}$$
$$= \alpha_i + \beta_{i,0} R_{m,t} + \beta_{i,1} Z_t R_{m,t} + \varepsilon_{i,t}$$

Now we have a multifactor model: easy to estimate and to test.

Testing the conditional CAPM: H_0: $\beta_{i,1}= 0$. (A t-test would do it.)

Note: An application of this example is the up-β and down-β:

$Z_t=1$ if $g(\mathbf{R}_{m,t-1}) >0$ -say, $g(\mathbf{R}_{m,t-1}) = R_{m,t-1}$
$Z_t=0$ otherwise.

Conditional vs. Unconditional CAPM

- The conditional CAPM says that expected returns are proportional to conditional betas: $E[R_{i,t}|I_{t-1}] = \beta_{i,t-1}\ \gamma_{t-1}$.

- Taking unconditional expectations:

$$E[R_{i,t}] = E[\beta_{i,t-1}]\ E[\gamma_{t-1}] + Cov(\gamma_{t-1},\beta_{i,t-1}) = \beta\ \gamma + Cov(\gamma_{t-1},\beta_{i,t-1})$$

- The asset's unconditional alpha is defined as:

$$\alpha^u \equiv E[R_{i,t}] - \beta^u\ \gamma$$

- Substituting for $E[R_{i,t}]$ yields:

$$\alpha^u = \gamma\ (\beta - \beta^u) + cov(\beta_{i,t-1},\ \gamma_{t-1}).$$

- Note: Under some conditions, discussed below, a stock's β^u and its expected conditional beta (β) will be similar.

- It can be shown (see Lewellen and Nagel (2006)):

$$\alpha^u = [1-\gamma^2/\sigma_m^2]cov(\beta_{t-1},\gamma_{t-1}) - \gamma/\sigma_m^2\ cov(\beta_{t-1},(\gamma_{t-1}-\gamma)^2) - \gamma/\sigma_m^2\ cov(\beta_{t-1},\sigma_{m,t}^2)$$

- Some implications:

  - It is well known that the conditional CAPM could hold perfectly, period-by-period, even though stocks are mispriced by the unconditional CAPM. Jensen (1968), Dybvig and Ross (1985), and Jagannathan and Wang (1996).

  - A stock's conditional alpha (or pricing error) might be zero, when its α^u is not, if its beta changes through time and is correlated with the equity premium or with conditional market volatility.

  - That is, the market portfolio might be conditionally MV efficient in every period but, at the same time, not on the unconditional MV efficient frontier. Hansen and Richard (1987).

Application 1: International CAPM

- From the CAPM DGP, the International CAPM can be written:

$$R_{i,t} = \alpha_i + \beta_i R_{w,t} + \varepsilon_{i,t}$$

$$\beta_i = Cov(R_{i,t}, R_{w,t})/Var(R_{w,t})$$

- Using a bivariate GARCH model, we can make β time varying:

$$\beta_{i,t} = Cov_t(R_{i,t}, R_{w,t})/Var_t(R_{w,t})$$

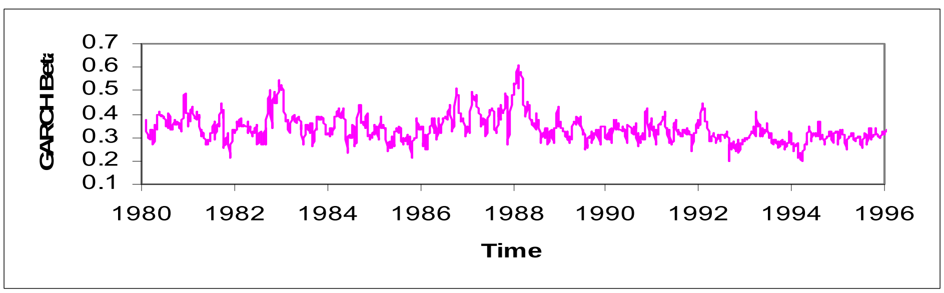
- A model for the World factor is needed. Usually, an AR(p) model:

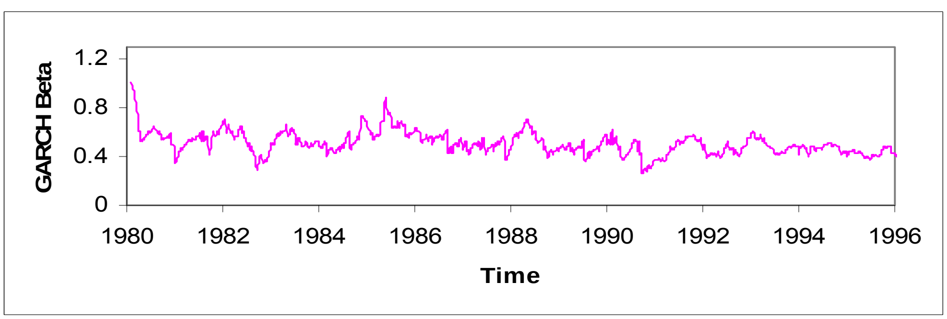
$$R_{w,t} = \delta_0 + \delta_1 R_{w,t-1} + \varepsilon_{w,t}$$

where $\varepsilon_{w,t}$ and $\varepsilon_{i,t}$ follow a bivariate GARCH model.

Mark (1988) and Ng (1991) find significant time-variation in $\beta_{i,t}$.

US and UK: World Beta-varying coefficients using bivariate GARCH





• *Braun, Nelson and Sunier* (1995): Use an E-GARCH framework, where $\beta_{i,t}$ also respond asymmetrically to positive versus negative domestic ($\varepsilon_{i,t}$) or world news ($\varepsilon_{w,t}$).

$$R_{i,t} = \alpha_i + \beta_i(\varepsilon_{i,t},\varepsilon_{w,t})\, R_{w,t}+\varepsilon_{i,t}$$

They find no significant time-variation evidence for their version of $\beta_{i,t}$.

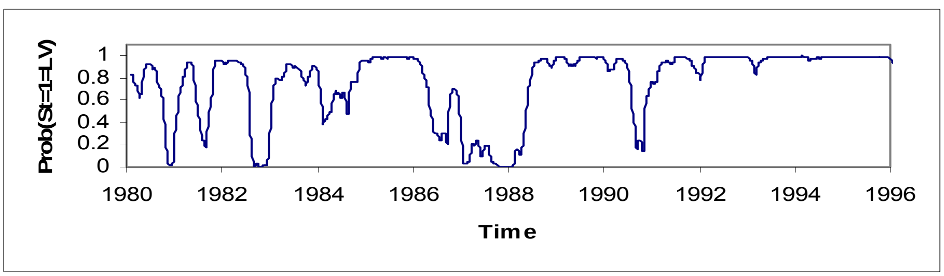
• *Ramchand and Susmel* (1998): use a SWARCH model, where $\beta_{i,t}$ is state dependent:

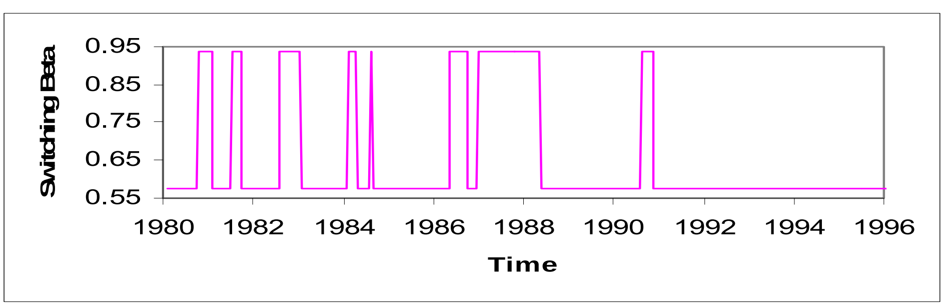
$$R_{i,t} = \alpha_i + (\beta_{i,0} + \beta_{i,1}\, S_t)\, R_{w,t}+ \varepsilon_{i,t},$$

where $\varepsilon_{i,t}$ follows a SWARCH model.

Strong evidence for state dependent $\beta_{i,t}$ in Pacific and North American markets, not that significant in European markets.

<u>US</u>: World Beta-varying coefficients using SWARCH model





• *Bekaert and Harvey* (1995): Study a conditional version of the ICAPM for emerging markets' stocks, where beta is conditioned on an unobservable state variable that takes on the value of zero or one.

$$R_{i,t} = \alpha + \beta_1 (1-S_t) R_{m,t-1} + \beta_2 S_t R_{w,t-1} + \varepsilon_{w,t}$$

where S_t is an unobservable state variable, which they considered linked to the degree of the emerging market's integration with a world benchmark.

They find evidence for time variation on β_1 and β_2, somewhat consistent with partial integration.

Note: These International CAPM papers do not use exogenous observable information. These papers focus on the time-series side of expected returns. They provide a very simple way of constructing time-varying betas.

Application 2: CS returns

- *Ferson and Harvey* (1993): Attempt to explain the CS expected returns across world stock markets.

- FH make $\alpha_{i,t}$ and $\beta_{i,t}$ linear function of variables such as dividend yields and the slope of the term structure.

$$R_{i,t} = (\alpha_{0i}+\alpha'_{1i}Z_{t-1}+\alpha'_{2i}A_{i,t-1}) + (\beta_{0i}+\beta'_{1i}Z_{t-1}+\beta'_{2i}A_{i,t-1}) R_{m,t}+\varepsilon_{i,t}$$

Z_{t-1}: global variables ("instruments") that affect all assets –say, interest rates, world and national factors.

$A_{i,t-1}$: asset specific variables ("instruments") –say, P/E, D/P, volatility.

Note: "Instruments," since they are pre-determined at t.

• FH find several instruments to be significant –i.e., $Cov(\gamma_{1,t-1},\beta_{i,t-1}) \neq 0$.

- Betas are time-varying, mostly due to local variables: E/P, inflation, long-term interest rates.
- Alphas are also time-varying, due to:E/P, P/CF, P/BV, volatility, inflation, long-term interest rates, and the term spread.
- Economic significance: typical abnormal return (in response to 1σ change in X) around 1-2% per month

Overall, however, the model explains a small percentage of the predicted time variation of stock returns.

Note: Ferson and Korajczyck (1995), though, using a similar model for the U.S. stock market, cannot reject the constant $ß_i$ model.

• *Jagannathan and Wang* (1996): Work with the SML to explain CS returns:

$$E[R_{i,t} - r_f] = E[\gamma_{0,t-1}] + E[\gamma_{1,t-1}]\, E[\beta_{i,t-1}] + Cov(\gamma_{1,t-1},\beta_{i,t-1})$$

• They decompose the conditional beta of any asset into 2 orthogonal components by projecting the conditional beta on the market risk premium.

- For each asset *i*, JW define the beta-premium sensitivity as

$$\upsilon_i = Cov(\gamma_{1,t-1},\beta_{i,t-1})/Var(\gamma_{1,t-1})$$

$$\eta_{i,t-1} = \beta_{i,t-1} - E[\beta_{i,t-1}] - \upsilon_i\,(\gamma_{1,t-1} - E[\gamma_{1,t-1}])$$

υ_i measures the sensitivity of the conditional beta to the market risk premium.

Then, rewriting the last equation as a regression:

$$\beta_{i,t-1} = E[\beta_{i,t-1}] - \upsilon_i\,(\gamma_{1,t-1} - E[\gamma_{1,t-1}]) + \eta_{i,t-1}$$

where $E[\eta_{i,t-1}] = E[\gamma_{1,t-1},\eta_{i,t-1}] = 0$.

• Now, the conditional beta can be written in three parts:

– The expected (unconditional) beta.

– A random variable perfectly correlated with the conditional market risk premium.

– Something mean zero and uncorrelated with the conditional market risk premium.

• Going back to the SML:

$$E[R_{i,t} - r_f] = E[\gamma_{0,t-1}] + E[\gamma_{1,t-1}]\, E[\beta_{i,t-1}] + \upsilon_i\, Var(\gamma_{1,t-1})$$

The unconditional expected return on any asset *i* is a linear function of

– Expected beta

– Beta-prem sensitivity, the larger the sensitivity, the larger the variability of the "second part" of the conditional beta.

Note: The beta-prem sensitivity measures instability of β_i over the business cycle. Stocks with β_i that vary more over the cycle have higher $E[R_{i,t} - r_f]$.

• We are back to the Fama-MacBeth (1973) CS estimation.

• To estimate the model, we need to estimate:

- Expected beta: $E[\beta_{i,t-1}]$

- Estimates of beta-prem sensitivity: υ_i.

• We can see η does not affect expected returns, it affect $\beta_{i,t-1}$. Thus, we can concentrate on the first two parts of the conditional beta.

• We need to make assumptions about the stochastic process governing the joint temporal evolution of $\beta_{i,t-1}$ and $\gamma_{1,t-1}$.

• Usually, the JW-type conditional CAPM is estimated using the following SML formulation:

$$E[R_{i,t} - r_f] = \gamma_0 + \gamma_1 E[\beta_{i,t-1}] + \lambda_i$$

where $E[\beta_{i,t-1}]$ will be an average beta for asset i and λ_i measures how the stock's beta co-varies though time with the risk premium. Different assumptions will deliver different $E[\beta_{i,t-1}]$ and λ_i.

• Findings: JW find that the betas of small, high-B/M stocks vary over the business cycle in a way that, according to JW, largely explains why those stocks have positive unconditional alphas.

• Lettau and Ludvigson (2001), Santos and Veronesi (2005), and Lustig and Van Nieuwerburgh (2005) find similar results. All papers find a dramatic increase in R^2 for their conditional models.

• *Lettau and Ludvigson* (2001): Estimate how a stock consumption betas change with the consumption-to-wealth ratio, or CAY:

$$\beta_{i,t} = \beta_i + \delta_i \, CAY_t$$

where β_i and δ_i are estimated in the first-pass regression:

$$R_{i,t} = \alpha_{i0} + \alpha_{i1} \, CAY_t + \beta_i \, \Delta c_t + \delta_i \, CAY_t \, \Delta c_t + \varepsilon_t,$$

CAY_t is the consumption residuals from a Stock and Watson (1993) cointegrating regression, with assets (a_t) and labor income (y_t):

$CAY_t = c_t - 0.31 \, a_t - 0.59 \, y_t - .60.$

Then, substituting $\beta_{i,t}$ into the unconditional relation gives:

$$E[R_{it}] = \beta_i \, \gamma + \delta_i \, cov(CAY_t, \gamma_t).$$

Note: There are some econometric issues here. Wealth (human capital) is not observable. Stationarity of proxy is an empirical matter.

• LL call their model a conditional C-CAPM. (More on Lecture 10.)

- LL use as γ a market returns and Δy_t or Δc_t to estimate the SML.

- They also include other variables in the SML to test their conditional C-CAPM: Size and B/M. (Traditional omitted variables test)

- Note: LL's model implies that the slope on β_i should be the average consumption-beta risk premium and the slope on δ_i should be $cov(CAY_t,\gamma_t)$.

- **Class comment**: Check the last row (6) on Table 6, Panel B –taken from LL. No coefficient has a significant t-stat, but R^2 is huge (.78)! Multicollinearity problem? (Recall that multicollinearity affects the standard errors, but not the estimates. The estimates are unbiased)

TABLE 6

Fama-MacBeth Regressions including Characteristics

A. λ_j Estimates on Betas in Cross-Sectional Regressions Including Size

		Factors$_{t+1}$			$\widehat{cay}_t$ · Factors$_{t+1}$				R^2
Row	Constant	R_{vw}	Δy	Δc	R_{vw}	Δy	Δc	Size	($\bar{R}^2$)
1	14.18	−3.60						−.57	.70
	(4.77)	(−2.78)						(−3.46)	.67
	(4.35)	(−2.54)						(−3.15)	
2	13.10	−3.06			.82			−.49	.75
	(4.71)	(−2.49)			(3.14)			(−3.24)	.73
	(3.79)	(−2.01)			(2.52)			(−2.61)	
3	12.03	−3.00	.51					−.41	.74
	(4.56)	(−2.52)	(2.00)					(−2.81)	.70
	(3.73)	(−2.06)	(1.63)					(−2.30)	
4	10.33	−2.68	.33		.59	−.02		−.33	.80
	(3.78)	(−2.33)	(1.36)		(2.63)	(−.59)		(−1.93)	.76
	(2.97)	(−1.84)	(1.07)		(2.07)	(−.46)		(−1.52)	
5	5.59			.04				−.18	.22
	(2.04)			(.35)				(−1.11)	.15
	(2.03)			(.35)				(−1.10)	
6	6.09			−.16			.08	−.15	.72
	(2.21)			(−1.45)			(3.23)	(−.87)	.68
	(1.66)			(−1.09)			(2.42)	(−.65)	

B. λ_j Estimates on Betas in Cross-Sectional Regressions Including Book-Market Ratio

		Factors$_{t+1}$			$\widehat{cay}_t$ · Factors$_{t+1}$			Book-Market	R^2
Row	Constant	R_{vw}	Δy	Δc	R_{vw}	Δy	Δc	Ratio	($\bar{R}^2$)
1	2.25	1.47						1.17	.82
	(2.06)	(1.08)						(3.62)	.81
	(2.01)	(1.06)						(3.57)	
2	2.22	1.45			.15			1.12	.83
	(2.01)	(1.05)			(.77)			(3.51)	.81
	(1.95)	(1.02)			(.75)			(3.41)	
3	1.91	2.00	.41					1.38	.83
	(1.68)	(1.41)	(1.61)					(3.89)	.80
	(1.52)	(1.29)	(1.44)					(3.53)	
4	2.81	.97	−.23		.14	−.05		1.09	.85
	(2.56)	(.71)	(−.94)		(.70)	(−1.56)		(3.13)	.81
	(2.36)	(.66)	(−.86)		(.64)	(−1.44)		(2.88)	
5	3.69			.14				.83	.75
	(5.98)			(.81)				(2.81)	.73
	(5.70)			(.77)				(2.67)	
6	3.90			.08			.02	.61	.78
	(6.29)			(.55)			(1.40)	(1.86)	.75
	(5.95)			(.52)			(1.32)	(1.75)	

Note.—See notes to tables 1–3. This table presents estimates of cross-sectional Fama-MacBeth regressions using the returns on 25 Fama-French portfolios:

$$E[R_{i,t+1}] = E[R_{0,t}] + \beta\lambda + \theta\Theta_i$$

where Θ_i denotes a characteristic variable: Θ_i is either the log of the portfolio size (size in panel A) or the log of the portfolio book-to-market ratio (in panel B).

Conditional CAPM: Does it Work?

- *Lewellen and Nagel* (2006): argue that variation in betas and the equity premium would have to be implausibly large to explain the asset pricing anomalies like momentum and the value premium.

- LN use a simple test of the conditional CAPM using direct estimates of conditional α and β from short-window regressions –i.e., assuming that α and β do not change in the estimation window. (Maybe, not a trivial assumption during some periods.)

- LN claim that they are avoiding the need to specify I_t.
- Fama and French (1993) methodology, adding momentum factor.
- LN estimate α and β quarterly, semiannually, and annually.

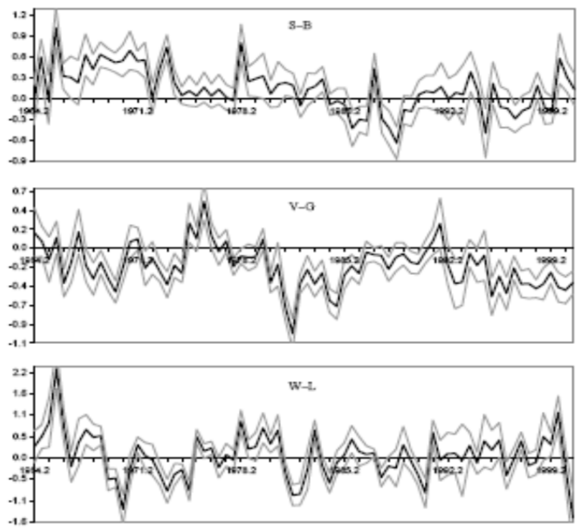
Table 3
Average conditional alphas, 1964 – 2001
The table reports average conditional alphas for size, B/M, and momentum portfolios (% monthly). Alphas are estimated quarterly using daily returns, semiannually using daily and weekly returns, and annually using monthly returns. The portfolios are formed from all NYSE and Amex stocks on CRSP / Compustat. We begin with 25 size-B/M portfolios (5×5 sort, breakpoints determined by NYSE quintiles) and 10 return-sorted portfolios, all value weighted. 'Small' is the average of the five low-market-cap portfolios, 'Big' is the average of the five high-market-cap portfolios, and 'S-B' is their difference. Similarly, 'Growth' is the average of the five low-B/M portfolios, 'Value' is the average of the five high-B/M portfolios, and 'V-G' is their difference. Return-sorted portfolios are formed based on past 6-month returns. 'Losers' is the bottom decile, 'Winners' is the top decile, and 'W-L' is their difference. Bold denotes estimates greater than two standard errors from zero.

	Size			B/M			Momentum		
	Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
Average conditional alpha (%)									
Quarterly	**0.42**	0.00	0.42	-0.01	**0.49**	**0.50**	**-0.79**	**0.55**	**1.33**
Semiannual 1	0.26	0.00	0.26	-0.08	**0.40**	**0.47**	**-0.61**	**0.39**	**0.99**
Semiannual 2	0.16	0.01	0.15	-0.12	**0.36**	**0.48**	**-0.83**	**0.53**	**1.37**
Annual	-0.06	0.08	-0.14	-0.20	0.32	**0.53**	**-0.56**	0.21	**0.77**
Standard error									
Quarterly	0.20	0.06	0.22	0.12	0.14	0.14	0.20	0.13	0.26
Semiannual 1	0.21	0.06	0.23	0.12	0.14	0.15	0.19	0.14	0.25
Semiannual 2	0.21	0.06	0.23	0.14	0.15	0.16	0.20	0.15	0.27
Annual	0.26	0.07	0.29	0.16	0.17	0.14	0.21	0.17	0.29

Quarterly and Semiannual 1 alphas are estimated from daily returns, Semiannual 2 alphas are estimated from weekly returns, and Annual alphas are estimated from monthly returns.

Figure 2
Conditional betas, 1964 – 2001

The figure plots conditional betas for size, B/M, and momentum portfolios. The dark line is the point estimate and the light lines indicate a two-standard-deviation confidence interval. Betas are estimated semiannually (non-overlapping windows) using daily returns. The portfolios are formed from all NYSE and Amex stocks on CRSP / Compustat. We begin with 25 size-B/M portfolios (5×5 sort, breakpoints determined by NYSE quintiles) and 10 return-sorted portfolios, all value weighted. 'S–B' is the average return on the five low-market-cap portfolios (Small) minus the average return on the five high-market-cap portfolios (Big). 'V–G' is the average return on the five high-B/M portfolios (Value) minus the average return on the five low-B/M portfolios (Growth). Return-sorted portfolios are formed based on past 6-month returns. 'W–L' is the return on the top decile (Winners) minus the return on the bottom decile (Losers).



• Findings: The conditional CAPM performs nearly as poorly as the unconditional CAPM.

- The conditional alphas (pricing errors) are significant.
- The conditional betas change over time. But, not enough to explain unconditional alphas. (Not enough co-variation with the market risk premium or volatility.)

• LN have a final good insight on Conditional CAPM tests:

- LN Conditional CAPM models estimate a restricted version of the SML, imposing a constraint on the slope of λ_i. The slope of λ_i is equal to 1:

$$E[R_{i,t} - r_f] = \gamma_0 + \gamma_1 E[\beta_{i,t-1}] + \lambda_i$$

In their tests, LN reject this restriction.